УДК 539.12

GENERAL ANALYSIS OF POLARIZATION EFFECTS IN THE REACTION $\gamma + \mathrm{d} \to \mathrm{d} + \pi + \pi$. II. SPIN CORRELATION COEFFICIENTS

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Received 1 October 2011

A general analysis of the polarization observables in the reaction of coherent photoproduction of a pair pseudoscalar mesons on the deuteron target, $\gamma + \mathrm{d} \to \mathrm{d} + \pi + \pi$, has been derived. This analysis does not depend on the details of the reaction mechanism since it is based on the general symmetry properties of the electromagnetic interaction with hadrons. Expressions for the spin correlation coefficients have been calculated in the terms of the reaction scalar amplitudes. These coefficients are caused by the linear or circular polarization of the photon beam and by the vector or tensor polarized deuteron target. The helicity amplitudes describing this reaction are also calculated. The experimental situation when scattered deuteron and one of the produced pions are detected in coincidence has been considered. The expressions for the spin correlation coefficients for the case of the reaction $\gamma + \mathrm{d} \to \mathrm{d} + \pi$ have been also derived. The helicity amplitudes describing the reaction $\gamma + \mathrm{d} \to \mathrm{d} + \pi$ are also calculated.
KEY WORDS: polarization, cross section, photoproduction, spin correlation, electron, deuteron.

ЗАГАЛЬНИЙ АНАЛІЗ ПОЛЯРИЗАЦІЙНИХ ЕФЕКТІВ У РЕАКЦІЇ $\gamma + \mathrm{d} \to \mathrm{d} + \pi + \pi$. II. КОЕФІЦІЄНТИ КОРЕЛЯЦІЇ СПІНІВ

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Виконано загальний аналіз виразів для поляризаційних спостережуваних у реакції когерентного фото утворення пари псевдоскалярних мезонів на дейтронній мішені, $\gamma + \mathrm{d} \to \mathrm{d} + \pi + \pi$. Цей аналіз не залежить від деталей механізму реакції тому, що він засновано на загальних властивостях симетрії електромагнітної взаємодії адронів. Вирахувані вирази для коефіцієнтів кореляції спінів у термінах скалярних амплітуд реакції. Ці коефіцієнти обумовлені лінійною або циркулярною поляризацією фотонного пучка та векторною або тензорною поляризацією дейтрона мішені. Також вирахувані спіральні амплітуди, які описують цю реакцію. Розглянута експериментальна постановка досліду, коли розсіюваний дейтрон і один із мезонів що утворюється детектуються на збіг. Вирахувані також вирази для коефіцієнтів кореляції спінів у реакції $\gamma + \mathrm{d} \to \mathrm{d} + \pi$. Вирахувані також спіральні амплітуди, які описують реакцію $\gamma + \mathrm{d} \to \mathrm{d} + \pi$.
КЛЮЧОВІ СЛОВА: поляризація, переріз, фотоутворення, кореляція спінів, електрон, дейтрон.

ОБЩИЙ АНАЛИЗ ПОЛЯРИЗАЦИОННЫХ ЭФФЕКТОВ В РЕАКЦИИ $\gamma + \mathrm{d} \to \mathrm{d} + \pi + \pi$. II. КОЭФФИЦИЕНТЫ КОРРЕЛЯЦИИ СПИНОВ

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Выполнен общий анализ выражений для поляризационных наблюдаемых в реакции когерентного фотообразования пары псевдоскалярных мезонов на дейтронной мишени, $\gamma + \mathrm{d} \to \mathrm{d} + \pi + \pi$. Этот анализ не зависит от деталей механизма реакции, так как он основан на общих свойствах симметрии электромагнитного взаимодействия адронов. Вычислены выражения для коэффициентов корреляции спинов в терминах скалярных амплитуд реакции. Эти коэффициенты обусловлены линейной или циркулярной поляризацией фотонного пучка и векторной или тензорной поляризацией дейтронной мишени. Также вычислены спиральные амплитуды, описывающие эту реакцию. Рассмотрена экспериментальная постановка опыта, когда рассеянный дейтрон и один из образующихся пионов детектируются на совпадения. Получены также выражения для коэффициентов корреляции спинов в реакции $\gamma + \mathrm{d} \to \mathrm{d} + \pi$. Вычислены также спиральные амплитуды, описывающие реакцию $\gamma + \mathrm{d} \to \mathrm{d} + \pi$.
КЛЮЧЕВЫЕ СЛОВА: поляризация, сечение, фоторождение, корреляция спинов, электрон, дейтрон.

Hadrons are complex systems of confined quarks and gluons and, therefore, exhibit the characteristic spectra of the hadron excited states. A thorough investigation of the hadron (and, in particular, nucleon) excitation spectrum is important problem for our understanding of quantum chromodynamics and of the effective degrees of freedom underlying hadronic matter.

As we noted in our previous paper [1], the various versions of the quark models is a source of the comprehensive predictions of the nucleon excitation spectrum. While many predicted properties of the nucleon resonances with low mass (< 1.8 GeV) agree fairly well with experimental measurements, there are discrepancies for the nucleon resonances with masses greater than this value. It is the well known problem of the "missing" resonances.

It was suggested that such resonances have small couplings with pion-nucleon state. So, to find them it is necessary to investigate the so-called multimeson final states.

The experimental investigation of the photoproduction processes has already proved its efficiency for the understanding of the excitation spectrum and the properties of baryons (see the review [2]). In comparison to single meson photoproduction the importance of the double-meson photoproduction increases at higher energies due to higher cross sections. The experiments on the meson photoproduction showed that double pion photoproduction is an important reaction channel in the second resonance region. The cross sections for single meson photoproduction (pion or η-meson) and double pion photoproduction are almost equal at the photon beam energies in the range 600 - 800 MeV [3,4].

Measurement of the differential cross sections alone lead to ambiguous sets of resonances contributing to a particular photoproduction channel since almost all information on interference effects is lost. So, it is necessary to use the measurement of various polarization observables. Current experimental efforts with the CLAS spectrometer at JLAB (USA) use highly-polarized frozen-spin targets in combination with polarized photon beams. The status of the recent double-polarization experiments and some preliminary results are discussed in Ref. [5].

A general analysis of the polarization observables in the reaction of coherent photoproduction of pair pseudoscalar mesons on the deuteron target, $\gamma d \to \pi\pi d$, has been derived in this paper. Our analysis does not depend on the details of the reaction mechanism since it is based on general symmetry properties of the electromagnetic interaction with hadrons. Expressions for the spin correlation coefficients have been calculated in terms of the reaction scalar amplitudes. These coefficients are caused by the the linear or circular polarization of the photon beam and by the vector or tensor polarization of the deuteron target. The helicity amplitudes describing this reaction are also calculated. The experimental situation when scattered deuteron and one of the produced pions are detected in coincidence has been considered. The expressions for the spin correlation coefficients for the case of the reaction $\gamma d \to \pi d$ have been also derived. The helicity amplitudes describing the reaction $\gamma d \to \pi d$ are calculated. In the Appendix A we give some formulae describing the polarization state of the deuteron target for different cases. In the Appendix B we present the expressions for the reaction scalar amplitudes in terms of the helicity amplitudes.

The aim of the paper is the analysis of the polarization observables in the reaction $\gamma \mathrm{d} \to \pi\pi \mathrm{d}$, which can help in elucidation of the reaction mechanism and clarify the properties of the "missing" resonances.

REACTION $\gamma + d \to d + \pi + \pi$

Let us consider the process of the photoproduction of a pair of the pseudoscalar mesons ($\pi\pi, \pi\eta$ and etc.) on the deuteron target

$$\gamma(k) + d(p_1) \to d(p_2) + \pi(q_1) + \pi(q_2), \tag{1}$$

where the four–momenta of the particles are given in the brackets. All notations which are absent in this paper can be found in Ref. [1].

The differential cross section of the reaction (1) in the experimental set-up when the scattered deuteron and one of the produced pions are detected in coincidence can be written as follows (for the case when all particles participating in the reaction are unpolarized)

$$\frac{d\sigma_{un}}{d\omega_1 d\Omega_\pi d\Omega_d} = KD, \qquad K = \frac{\alpha}{3(4\pi)^4}\frac{p^2\,|\vec{q}_1|}{s - M^2}[p(W-\omega_1) + E_2\,|\vec{q}_1|\,cos\chi]^{-1}, \tag{2}$$

where $S = W^2 = (k + p_1)^2$ is the square of the reaction total energy, M is the deuteron mass, $\omega_1(|\vec{q}_1|)$ and $E_2(p)$ are the energy (magnitude of the momentum) of the first meson and scattered deuteron in the reaction CMS, χ is the angle between momenta of the scattered deuteron and the first meson. K is the kinematical factor and the quantity D is

$$D = x_1[|f_1|^2 + |f_8|^2 + |g_{12}|^2 + |g_{14}|^2 + \gamma_1^2(|f_5|^2 + |g_{18}|^2)] + x_2[|f_2|^2 + |f_6|^2 + |g_{10}|^2 + \\ + |g_{15}|^2 + \gamma_1^2(|f_7|^2 + |g_{11}|^2)] + x_3[|f_4|^2 + |f_9|^2 + |g_{13}|^2 + |g_{17}|^2 + \gamma_1^2(|f_3|^2 + |g_{16}|^2)] + \tag{3}$$

$$+2y_1\,\mathrm{Re}[f_1g_{10}^* + f_2g_{12}^* + f_6g_{14}^* + f_8g_{15}^* + \gamma_1^2(f_5g_{11}^* + f_7g_{18}^*)] + 2y_2\,\mathrm{Re}[f_1f_4^* + f_8f_9^* + g_{12}g_{13}^* +$$
$$+g_{14}g_{17}^* + \gamma_1^2(f_3f_5^* + g_{16}g_{18}^*)] + 2y_3\,\mathrm{Re}[f_2g_{13}^* + f_4g_{10}^* + f_6g_{17}^* + f_9g_{15}^* + \gamma_1^2(f_3g_{11}^* + f_7g_{16}^*)],$$

where quantities $f_i\ (i=1-9)$ and $g_i\ (i=10-18)$ are the scalar independent amplitudes for the reaction (1) which are functions of the five kinematical variables (the definition of these amplitudes is given in Ref. [1]). We introduce the following designations

$$x_1 = 1+\frac{p^2}{M^2}\sin^2\theta\cos^2\varphi,\quad x_2 = 1+\frac{p^2}{M^2}\sin^2\theta\sin^2\varphi,\quad x_3 = 1+\frac{p^2}{M^2}\cos^2\theta,$$
$$y_1 = \frac{p^2}{2M^2}\sin^2\theta\sin 2\varphi,\quad y_2 = \frac{p^2}{2M^2}\sin 2\theta\cos\varphi,\quad y_3 = \frac{p^2}{2M^2}\sin 2\theta\sin\varphi,\quad \gamma_1 = \frac{E_1}{M},$$

where θ and φ are the polar and azimuthal angles of the recoil deuteron momentum, E_1 is the energy of the initial deuteron in the reaction CMS.

1. The photon beam is elliptically polarized and deuteron target is vector polarized.

The part of the differential cross section of the reaction (1) which proportional to the spin correlation coefficients can be written as follows for the case when the photon beam has an elliptical polarization and the deuteron target is vector polarized

$$\frac{d\sigma}{d\omega_1 d\Omega_\pi\, d\Omega_d} = \frac{d\sigma_{un}}{d\omega_1 d\Omega_\pi\, d\Omega_d}[1+\cos 2\beta(C_x^L\xi_x + C_y^L\xi_y + C_z^L\xi_z) +$$
$$+\sin 2\beta\cos\delta(\bar{C}_x^L\xi_x + \bar{C}_y^L\xi_y + \bar{C}_z^L\xi_z) + \sin 2\beta\sin\delta(C_x^c\xi_x + C_y^c\xi_y + C_z^c\xi_z)], \qquad (4)$$

where $\vec{\xi}$ is the unit vector of the deuteron vector polarization in its rest system and the quantities C_i^L, $\bar{C}_i^L$ and (C_i^c), $i = x, y, z$, are the spin correlation coefficients caused by the vector polarization of the initial deuteron provided the photon beam is linearly (circularly) polarized. These polarization observables have the following expressions in terms of the reaction scalar amplitudes

$$DC_x^L = 3\gamma_1\,\mathrm{Im}[x_1(f_8g_{18}^* - f_5g_{12}^*) + x_2(f_2g_{11}^* - f_7g_{15}^*) + x_3(f_9g_{16}^* - f_3g_3^*) + y_1(f_2f_5^* - f_7f_8^* +$$
$$+g_{15}g_{18}^* - g_{11}g_{12}^*) + y_2(f_8g_{16}^* + f_9g_{18}^* - f_3g_{12}^* - f_5g_{13}^*) + y_3(f_2f_3^* - f_7f_9^* + g_{15}g_{16}^* - g_{11}g_{13}^*)], \qquad (5)$$

$$DC_y^L = 3\gamma_1\,\mathrm{Im}[x_1(g_{18}g_{14}^* - f_1f_5^*) - x_2(f_6f_7^* + g_{10}g_{11}^*) + x_3(f_3f_4^* + g_{16}g_{17}^*) + y_1(f_5g_{10}^* + f_7g_{14}^* -$$
$$-f_6g_{18}^* - f_1g_{11}^*) - y_2(f_1f_3^* + f_4f_5^* + g_{14}g_{16}^* + g_{17}g_{18}^*) + y_3(f_3g_{10}^* + f_7g_{17}^* - f_4g_{11}^* - f_6g_{16}^*)], \qquad (6)$$

$$DC_z^L = 3\,\mathrm{Im}[x_1(f_1g_{12}^* - f_8g_{14}^*) + x_2(f_6g_{15}^* - f_2g_{10}^*) + x_3(f_4g_{13}^* - f_9g_{17}^*) + y_1(f_1f_2^* + f_6f_8^* +$$
$$+g_{10}g_{12}^* + g_{14}g_{15}^*) + y_2(f_1g_{13}^* + f_4g_{12}^* - f_8g_{17}^* - f_9g_{14}^*) + y_3(f_6f_9^* - f_2f_4^* + g_{10}g_{13}^* - g_{15}g_{17}^*)], \qquad (7)$$

$$D\bar{C}_x^L = 3\gamma_1\,\mathrm{Im}[x_1(g_{12}g_{18}^* - f_5f_8^*) + x_2(f_2f_7^* - g_{11}g_{15}^*) + x_3(g_{13}g_{16}^* - f_3f_9^*) + y_1(f_2g_{18}^* - f_5g_{15}^* -$$
$$-f_7g_{12}^* + f_8g_{11}^*) + y_2(g_{12}g_{16}^* + g_{13}g_{18}^* - f_3f_8^* - f_5f_9^*) + y_3(f_2g_{16}^* - f_7g_{13}^* - f_3g_{15}^* + f_9g_{11}^*)], \qquad (8)$$

$$D\bar{C}_y^L = 3\gamma_1\,\mathrm{Im}[x_1(f_5g_{14}^* - f_1g_{18}^*) + x_2(f_7g_{10}^* - f_6g_{11}^*) + x_3(f_3g_{17}^* - f_4g_{16}^*) + y_1(f_5f_6^* - f_1f_7^* +$$
$$+g_{11}g_{14}^* - g_{10}g_{18}^*) + y_2(f_3g_{14}^* - f_1g_{16}^* - f_4g_{18}^* + f_5g_{17}^*) + y_3(f_3f_6^* - f_4f_7^* + g_{11}g_{17}^* - g_{10}g_{16}^*)], \qquad (9)$$

$$D\bar{C}_z^L = 3\,\mathrm{Im}[x_1(f_1f_8^* - g_{12}g_{14}^*) + x_2(g_{10}g_{15}^* - f_2f_6^*) + x_3(f_4f_9^* - g_{13}g_{17}^*) + y_1(f_1g_{15}^* - f_8g_{10}^* -\ )$$
$$-f_2g_{14}^* + f_6g_{12}^*) + y_2(f_1f_9^* + f_4f_8^* - g_{12}g_{17}^* - g_{13}g_{14}^*) + y_3(f_4g_{15}^* - f_9g_{10}^* - f_2g_{17}^* + f_6g_{13}^*)], \qquad (10)$$

$$DC_x^c = -3\gamma_1\,\mathrm{Re}[x_1(g_{12}g_{18}^* - f_5f_8^*) + x_2(f_2f_7^* - g_{11}g_{15}^*) + x_3(g_{13}g_{16}^* - f_3f_9^*) + y_1(f_2g_{18}^* - f_5g_{15}^* +$$
$$+f_7g_{12}^* - f_8g_{11}^*) + y_2(g_{12}g_{16}^* + g_{13}g_{18}^* - f_3f_8^* - f_5f_9^*) + y_3(f_2g_{16}^* + f_7g_{13}^* - f_3g_{15}^* - f_9g_{11}^*)], \qquad (11)$$

$$DC_y^c = -3\gamma_1 \operatorname{Re}[x_1(f_5 g_{14}^* - f_1 g_{18}^*) - x_2(f_7 g_{10}^* - f_6 g_{11}^*) + x_3(f_3 g_{17}^* - f_4 g_{16}^*) + y_1(f_5 f_6^* - f_1 f_7^* + \\ + g_{11} g_{14}^* - g_{10} g_{18}^*) + y_2(f_3 g_{14}^* - f_1 g_{16}^* - f_4 g_{18}^* + f_5 g_{17}^*) + y_3(f_3 f_6^* - f_4 f_7^* + g_{11} g_{17}^* - g_{10} g_{16}^*)], \quad (12)$$

$$DC_z^c = -3 \operatorname{Re}[x_1(f_1 f_8^* - g_{12} g_{14}^*) + x_2(g_{10} g_{15}^* - f_2 f_6^*) + x_3(f_4 f_9^* - g_{13} g_{17}^*) + y_1(f_1 g_{15}^* + f_8 g_{10}^* - \\ - f_2 g_{14}^* - f_6 g_{12}^*) + y_2(f_1 f_9^* + f_4 f_8^* - g_{12} g_{17}^* - g_{13} g_{14}^*) + y_3(f_4 g_{15}^* + f_9 g_{10}^* - f_2 g_{17}^* - f_6 g_{13}^*)]. \quad (13)$$

From these expressions one can see that there are 9 spin correlation coefficients for the case of the vector polarized deuteron target instead of 5 ones for the case of the $\gamma + d \to d + \pi$ reaction. The spin correlation coefficients $C_i^L \left(\bar{C}_i^L\right)$, $i = x, y, z$ can be determined using the photon beam with linear polarization at an angle $\beta = 0^\circ$ and 90° ($\beta = 45^\circ$ and 135°). Using the circularly polarized photon beam allows to determine the spin correlation coefficients $C_i^c, i = x, y, z$.

Note that real amplitudes (the amplitudes in the impulse approximation for this reaction) lead to zero values of the following spin correlation coefficients $C_i^L, \bar{C}_i^L$, $i = x, y, z$, i.e., when the photon beam is linear polarized. In the case of the circularly polarized photon beam, the rest spin correlation coefficients may be non-zero ones.

2. The photon beam is elliptically polarized and deuteron target is tensor polarized.

The part of the differential cross section of the reaction (1) which proportional to the spin correlation coefficients can be written as follows for the case when the photon beam has an elliptical polarization and the deuteron target is tensor polarized

$$\frac{d\sigma}{d\omega_1 d\Omega_\pi \, d\Omega_d} = \frac{d\sigma_{un}}{d\omega_1 d\Omega_\pi \, d\Omega_d}\{1 + \cos 2\beta [C_{xx}^L (Q_{xx} - Q_{yy}) + C_{zz}^L Q_{zz} + C_{xz}^L Q_{xz} + C_{xy}^L Q_{xy} + C_{yz}^L Q_{yz}] + \\ + \sin 2\beta \cos\delta [\bar{C}_{xx}^L (Q_{xx} - Q_{yy}) + \bar{C}_{zz}^L Q_{zz} + \bar{C}_{xz}^L Q_{xz} + \bar{C}_{xy}^L Q_{xy} + \bar{C}_{yz}^L Q_{yz}] + \\ + \sin 2\beta \sin\delta [C_{xx}^c (Q_{xx} - Q_{yy}) + C_{zz}^c Q_{zz} + C_{xz}^c Q_{xz} + C_{xy}^c Q_{xy} + C_{yz}^c Q_{yz}]\}, \quad (14)$$

where $Q_{ij}, ij = xx, yy, zz, xy, xz, yz$ is the symmetrical tensor describing the tensor polarization of the deuteron target in the reaction CMS and the quantities C_{ij}^L and $\bar{C}_{ij}^L \left(C_{ij}^c\right)$, $i = xx, zz, xz, xy, yz$, are the spin correlation coefficients caused by the tensor polarization of the initial deuteron provided the photon beam is linearly (circularly) polarized. These polarization observables have the following expressions in terms of the reaction scalar amplitudes

$$DC_{xx}^L = \frac{3}{2}[x_1(|f_1|^2 + |f_8|^2 - |g_{12}|^2 - |g_{14}|^2) + x_2(|g_{10}|^2 + |g_{15}|^2 - |f_2|^2 - |f_6|^2) + \\ + x_3(|f_4|^2 + |f_9|^2 - |g_{13}|^2 - |g_{17}|^2) + 2y_1 Re(f_1 g_{10}^* + f_8 g_{15}^* - f_2 g_{12}^* - f_6 g_{14}^*) + \\ + 2y_2 Re(f_1 f_4^* + f_8 f_9^* - g_{12} g_{13}^* - g_{14} g_{17}^*) + 2y_3 Re(f_4 g_{10}^* + f_9 g_{15}^* - f_2 g_{13}^* - f_6 g_{17}^*)], \quad (15)$$

$$DC_{zz}^L = 3\{x_1(|f_5|^2 - |g_{18}|^2) + x_2(|g_{11}|^2 - |f_7|^2) + x_3(|f_3|^2 - |g_{16}|^2) + 2y_1 Re(f_5 g_{11}^* - f_7 g_{18}^*) + 2y_2 Re(f_3 f_5^* - g_{16} g_{18}^*) + \\ + 2y_3 Re(f_3 g_{11}^* - f_7 g_{16}^*) + \frac{1}{2\gamma_1^2}\Big[x_1(|f_8|^2 + |g_{14}|^2 - |f_1|^2 - |g_{12}|^2) + x_2(|f_6|^2 + |g_{15}|^2 - |f_2|^2 - |g_{10}|^2) + \\ + x_3(|f_9|^2 + |g_{17}|^2 - |f_4|^2 - |g_{13}|^2) + 2y_1 Re(f_6 g_{14}^* + f_8 g_{15}^* - f_2 g_{12}^* - f_1 g_{10}^*) + 2y_2 Re(f_8 f_9^* + g_{14} g_{17}^* - f_1 f_4^* - g_{12} g_{13}^*) + \\ + 2y_3 Re(f_6 g_{17}^* + f_9 g_{15}^* - f_2 g_{13}^* - f_4 g_{10}^*)\Big]\}, \quad (16)$$

$$DC_{xy}^L = 6 \operatorname{Re}[x_1(f_1 g_{12}^* - f_8 g_{14}^*) + x_2(f_2 g_{10}^* - f_6 g_{15}^*) + x_3(f_4 g_{13}^* - f_9 g_{17}^*) + y_1(f_1 f_2^* + g_{10} g_{12}^* - \\ - f_6 f_8^* - g_{14} g_{15}^*) + y_2(f_1 g_{13}^* + f_4 g_{12}^* - f_8 g_{17}^* - f_9 g_{14}^*) + y_3(f_2 f_4^* + g_{10} g_{13}^* - f_6 f_9^* - g_{15} g_{17}^*)], \quad (17)$$

$$DC_{xz}^L = 6 \operatorname{Re}[x_1(f_1 f_5^* - g_{14} g_{18}^*) + x_2(g_{10} g_{11}^* - f_6 f_7^*) + x_3(f_3 f_4^* - g_{16} g_{17}^*) + y_1(f_5 g_{10}^* + f_1 g_{11}^* - \\ - f_6 g_{18}^* - f_7 g_{14}^*) + y_2(f_1 f_3^* + f_4 f_5^* - g_{14} g_{16}^* - g_{17} g_{18}^*) + y_3(f_3 g_{10}^* + f_4 g_{11}^* - f_6 g_{16}^* - f_7 g_{17}^*)], \quad (18)$$

$$DC_{yz}^L = 6Re[x_1(f_5 g_{12}^* - f_8 g_{18}^*) + x_2(f_2 g_{11}^* - f_7 g_{15}^*) + x_3(f_3 g_{13}^* - f_9 g_{16}^*) + y_1(f_2 f_5^* + g_{11} g_{12}^* - \\ - f_7 f_8^* - g_{15} g_{18}^*) + y_2(f_3 g_{12}^* + f_5 g_{13}^* - f_8 g_{16}^* - f_9 g_{18}^*) + y_3(f_2 f_3^* + g_{11} g_{13}^* - f_7 f_9^* - g_{15} g_{16}^*)], \quad (19)$$

$$D\bar{C}^L_{xx} = 6\,\mathrm{Re}\,a_{xx}, \qquad DC^c_{xx} = 6\,\mathrm{Im}\,a_{xx},$$
$$a_{xx} = \frac{1}{2}[x_1(f_1g^*_{14} - g_{12}f^*_8) + x_2(g_{10}f^*_6 - f_2g^*_{15}) + x_3(f_4g^*_{17} - g_{13}f^*_9) + y_1(f_1f^*_6 + g_{10}g^*_{14} - \tag{20}$$
$$-f_2f^*_8 - g_{12}g^*_{15}) + y_2(f_1g^*_{17} + f_4g^*_{14} - g_{12}f^*_9 - g_{13}f^*_8) + y_3(f_4f^*_6 + g_{10}g^*_{17} - f_2f^*_9 - g_{13}g^*_{15})],$$

$$D\bar{C}^L_{zz} = 6\,\mathrm{Re}\,a_{zz}, \qquad DC^c_{zz} = 6\,\mathrm{Im}\,a_{zz}, \tag{21}$$
$$a_{zz} = x_1f_5g^*_{18} + x_2g_{11}f^*_7 + x_3f_3g^*_{16} + y_1(f_5f^*_7 + g_{11}g^*_{18}) + y_2(f_3g^*_{18} + f_5g^*_{16}) + y_3(f_3f^*_7 + g_{11}g^*_{16}) -$$
$$-\frac{1}{2\gamma_1^2}[x_1(f_1g^*_{14} + g_{12}f^*_8) + x_2(g_{10}f^*_6 + f_2g^*_{15}) + x_3(f_4g^*_{17} + g_{13}f^*_9) + y_1(f_1f^*_6 + g_{10}g^*_{14} + f_2f^*_8 + g_{12}g^*_{15}) +$$
$$+ y_2(f_1g^*_{17} + f_4g^*_{14} + g_{12}f^*_9 + g_{13}f^*_8) + y_3(f_4f^*_6 + g_{10}g^*_{17} + f_2f^*_9 + g_{13}g^*_{15})],$$

$$D\bar{C}^L_{xy} = 6\,\mathrm{Re}\,a_{xy}, \qquad DC^c_{xy} = 6\,\mathrm{Im}\,a_{xy},$$
$$a_{xy} = x_1(f_1f^*_8 + g_{12}g^*_{14}) + x_2(g_{10}g^*_{15} + f_2f^*_6) + x_3(f_4f^*_9 + g_{13}g^*_{17}) + y_1(f_1g^*_{15} + f_2g^*_{14} + \tag{22}$$
$$+g_{10}f^*_8 + g_{12}f^*_6) + y_2(f_1f^*_9 + f_4f^*_8 + g_{12}g^*_{17} + g_{13}g^*_{14}) + y_3(f_2g^*_{17} + f_4g^*_{15} + g_{10}f^*_9 + g_{13}f^*_6),$$

$$D\bar{C}^L_{xz} = 6\,\mathrm{Re}\,a_{xz}, \qquad DC^c_{xz} = 6\,\mathrm{Im}\,a_{xz}, \tag{23}$$
$$a_{xz} = x_1(f_1g^*_{18} + f_5g^*_{14}) + x_2(g_{10}f^*_7 + g_{11}f^*_6) + x_3(f_3g^*_{17} + f_4g^*_{16}) + y_1(f_1f^*_7 + g_{10}g^*_{18} +$$
$$+f_5f^*_6 + g_{11}g^*_{14}) + y_2(f_1g^*_{16} + f_3g^*_{14} + f_4g^*_{18} + f_5g^*_{17}) + y_3(f_3f^*_6 + g_{10}g^*_{16} + f_4f^*_7 + g_{11}g^*_{17}),$$

$$D\bar{C}^L_{yz} = 6\,\mathrm{Re}\,a_{yz}, \qquad DC^c_{yz} = 6\,\mathrm{Im}\,a_{yz}, \tag{24}$$
$$a_{yz} = x_1(f_5f^*_8 + g_{12}g^*_{18}) + x_2(g_{11}g^*_{15} + f_2f^*_7) + x_3(f_3f^*_9 + g_{13}g^*_{16}) + y_1(f_2g^*_{18} + f_5g^*_{15} +$$
$$+g_{11}f^*_8 + g_{12}f^*_7) + y_2(f_3f^*_8 + f_5f^*_9 + g_{12}g^*_{16} + g_{13}g^*_{18}) + y_3(f_2g^*_{16} + f_3g^*_{15} + g_{11}f^*_9 + g_{13}f^*_7).$$

From these expressions one can see that there are 15 spin correlation coefficients for the case of the tensor polarized deuteron target instead of 7 ones for the case of the $\gamma + d \to d + \pi$ reaction. The spin correlation coefficients $C^L_{ij}(\bar{C}^L_{ij})$, $ij = xx, zz, xz, xy, yz$ can be determined using the photon beam with linear polarization at an angle $\beta = 0^\circ$ and 90° ($\beta = 45^\circ$ and 135°). Using the circularly polarized photon beam allows to determine the spin correlation coefficients C^c_{ij}, $ij = xx, zz, xz, xy, yz$.

Note that real amplitudes (the amplitudes in the impulse approximation for this reaction) lead to zero values of the following spin correlation coefficients C^c_{ij}, $ij = xx, zz, xz, xy, yz$, i.e., when the photon beam is circularly polarized. In the case of the linearly polarized photon beam, the rest spin correlation coefficients may be non-zero ones.

HELICITY AMPLITUDES

This reaction, as any other reaction, may be described either by the scalar amplitudes or by the helicity ones. In the last case the axis of quantization of the spin of each particle is the direction of its momentum. The helicity states of the photon are (it moves along z axis)

$$e^{(\pm 1)}_\mu = \frac{1}{\sqrt{2}}(0, \mp 1, -i, 0). \tag{25}$$

The momenta of the particles in the reaction CMS are chosen as

$$k = (\omega, \mathbf{k}), \quad p_1 = (E_1, -\mathbf{k}), \quad q_1 = (\omega_1, \mathbf{q}), \quad q_2 = (\omega_2, \mathbf{q}'), \quad p_2 = (E_2, \mathbf{p}). \tag{26}$$

The z axis is directed along the photon momentum $\mathbf{k}$, the momentum $\mathbf{q}$ lies in the xz plane and the momentum of the final (scattered) deuteron is determined by the angles θ and φ. The y axis is directed along the vector $\mathbf{k} \times \mathbf{q}$, and x axis coincides with the direction $(\mathbf{k} \times \mathbf{q}) \times \mathbf{k}$. The helicity states of the initial deuteron are

$$U_{1\mu}^{(\pm1)} = \frac{1}{\sqrt{2}}(0, \pm 1, -i, 0), \qquad U_{1\mu}^{(0)} = \frac{1}{M}(-k, 0, 0, E_1). \tag{27}$$

Here we take into account the phase agreement for the particle moving along the negative direction of the z axis. The helicity states of the final deuteron can be written as

$$U_{2\mu}^{(\pm1)} = \frac{1}{\sqrt{2}}(0, \mp\cos\theta\cos\varphi + i\sin\varphi, \mp\cos\theta\sin\varphi - i\cos\varphi, \pm\sin\theta), \tag{28}$$

$$U_{2\mu}^{(0)} = \frac{1}{M}(p, E_2\sin\theta\cos\varphi, E_2\sin\theta\sin\varphi, E_2\cos\theta).$$

The matrix element of the reaction (1), with three-body final state, results in specific properties of the helicity amplitudes describing this process $H_{\lambda_d'}^{\lambda_\gamma\lambda_d}$, where λ_γ and $\lambda_d(\lambda_d')$ are the helicities of the photon and initial (recoil) deuteron. In the case of the binary reaction (for example, the reaction $\gamma + d \to d + \pi$), the P invariance of the electromagnetic and strong interactions implies that $|H_{-\lambda_d'}^{-\lambda_\gamma-\lambda_d}| = |H_{\lambda_d'}^{\lambda_\gamma\lambda_d}|$ [6]. But for the non-coplanar process (as the reaction (1)) the helicity amplitudes with opposite sign of the helicities are different ones. Therefore, the number of the independent helicity amplitudes for the reaction (1) is equal to $2s_\gamma(2s_d+1)(2s_d+1)=18$, where s_γ and s_d are the spin of the real photon and deuteron, respectively. Naturally, this number coincides with the number of the independent scalar amplitudes. The number of the independent helicity amplitudes for the reaction $\gamma + d \to d + \pi$ (binary collision) is twice smaller.

Let us calculate the helicity amplitudes of the reaction (1). We introduce the set of the helicity amplitudes $H_{\lambda_d'}^{\lambda_\gamma\lambda_d}$. As it is shown above, the matrix element of the reaction (1) can be described in terms of the scalar amplitudes. The relations between the helicity amplitudes and the scalar amplitudes are the following

$$H_{1,2} = H_{+}^{\pm+} = \mp\frac{e}{2\sqrt{2}}[\sin\theta(f_4 - ig_{13} \pm f_9 \pm ig_{17}) - (\cos\theta\cos\varphi + i\sin\varphi)(f_1 - ig_{12} \pm f_8 \pm ig_{14}) + \\ +(-\cos\theta\sin\varphi + i\cos\varphi)(g_{10} - if_2 \pm g_{15} \pm if_6)], \tag{29}$$

$$H_{3,4} = H_{-}^{\pm-} = \mp\frac{e}{2\sqrt{2}}[\sin\theta(f_4 + ig_{13} \mp f_9 \pm ig_{17}) - (\cos\theta\cos\varphi - i\sin\varphi)(f_1 + ig_{12} \mp f_8 \pm ig_{14}) - \\ -(\cos\theta\sin\varphi + i\cos\varphi)(g_{10} + if_2 \mp g_{15} \pm if_6)], \tag{30}$$

$$H_{5,6} = H_{0}^{\pm0} = \mp\frac{e}{\sqrt{2}}\frac{E_1E_2}{M^2}[\cos\theta(f_3 \pm ig_{16}) + \sin\theta\cos\varphi(f_5 \pm ig_{18}) + \sin\theta\sin\varphi(g_{11} \pm if_7)], \tag{31}$$

$$H_{7,8} = H_{-}^{\pm+} = \mp\frac{e}{2\sqrt{2}}[\sin\theta(ig_{13} - f_4 \mp f_9 \mp ig_{17}) + (\cos\theta\cos\varphi - i\sin\varphi)(f_1 - ig_{12} \pm f_8 \pm ig_{14}) + \\ +(\cos\theta\sin\varphi + i\cos\varphi)(g_{10} - if_2 \pm g_{15} \pm if_6)], \tag{32}$$

$$H_{9,10} = H_{+}^{\pm0} = \mp\frac{e}{2}\frac{E_1}{M}[\sin\theta(f_3 \pm ig_{16}) - (\cos\theta\cos\varphi + i\sin\varphi)(f_5 \pm ig_{18}) - (\cos\theta\sin\varphi - i\cos\varphi)(g_{11} \pm if_7)], \tag{33}$$

$$H_{11,12} = H_{-}^{\pm0} = \mp\frac{e}{2}\frac{E_1}{M}[-\sin\theta(f_3 \pm ig_{16}) + (\cos\theta\cos\varphi - i\sin\varphi)(f_5 \pm ig_{18}) + (\cos\theta\sin\varphi + i\cos\varphi)(g_{11} \pm if_7)], \tag{34}$$

$$H_{13,14} = H_{0}^{\pm+} = \mp\frac{e}{2}\frac{E_2}{M}[\cos\theta(f_4 - ig_{13} \pm ig_{17} \pm f_9) + \sin\theta\cos\varphi(f_1 - ig_{12} \pm ig_{14} \pm f_8) + \sin\theta\sin\varphi(g_{10} - if_2 \pm if_6 \pm g_{15})], \tag{35}$$

$$H_{15,16} = H_{0}^{\pm-} = \pm\frac{e}{2}\frac{E_2}{M}[\cos\theta(f_4 + ig_{13} \pm ig_{17} \mp f_9) + \sin\theta\cos\varphi(f_1 + ig_{12} \pm ig_{14} \mp f_8) + \sin\theta\sin\varphi(g_{10} + if_2 \pm if_6 \mp g_{15})], \tag{36}$$

$$H_{17,18} = H_{+}^{\pm-} = \mp\frac{e}{2\sqrt{2}}[-\sin\theta(ig_{13} + f_4 \mp f_9 \pm ig_{17}) + (\cos\theta\cos\varphi + i\sin\varphi)(f_1 + ig_{12} \mp f_8 \pm ig_{14}) + \\ +(\cos\theta\sin\varphi - i\cos\varphi)(g_{10} + if_2 \mp g_{15} \pm if_6)]. \quad (37)$$

The expressions for the reaction scalar amplitudes $f_i (i = 1-9)$ and $g_i (i = 10-18)$ in terms of the helicity amplitudes $H_i (i = 1-18)$ are given in the Appendix B.

REACTION $\gamma + d \to d + \pi$

In this section we apply at first the formalism of parametrization of the reaction amplitude with the help of the orthonormal basis to the case of the coherent photoproduction of a pion (or η–meson) on the deuteron

$$\gamma(k) + d(p_1) \to d(p_2) + \pi(q), \quad (38)$$

where the four–momenta of the particles are given in the brackets.

The coordinate frame in the reaction CMS is chosen as: *z* axis is directed along the momentum of the photon $\mathbf{k}$, and the momentum of the pion $\mathbf{q}$ lies in the *xz* plane, *y* axis is directed along the vector $\mathbf{k}\times\mathbf{q}$.

The differential cross section of this reaction for the case of unpolarized particles can be written as

$$\frac{d\sigma_{un}}{d\Omega} = NG, \qquad N = \frac{\alpha}{96\pi W^2}\frac{q}{\omega}, \quad (39)$$

where ω is the energy of the photon beam in the reaction CMS and the quantity *G* is

$$G = |g_1|^2 + |g_6|^2 + a_1(|g_3|^2 + |g_5|^2) + a_2(|g_4|^2 + |g_8|^2) + 2a_3\mathrm{Re}(g_3 g_4^* + g_5 g_8^*) + \\ + \gamma_1^2[|g_2|^2 + a_1|g_9|^2 + a_2|g_7|^2 + 2a_3\mathrm{Re}(g_7 g_9^*)], \\ a_1 = 1 + \frac{\mathbf{q}^2}{M^2}\sin^2\vartheta, \qquad a_2 = 1 + \frac{\mathbf{q}^2}{M^2}\cos^2\vartheta, \qquad a_3 = \frac{\mathbf{q}^2}{2M^2}\sin 2\vartheta, \quad (40)$$

where ϑ is the angle between the photon and pion momenta in the reaction CMS.

1. The deuteron target is vector polarized.

The differential cross section of this reaction for the case of the vector polarized initial deuteron and elliptically polarized photon beam can be written as

$$\frac{d\sigma_v}{d\Omega} = \frac{d\sigma_{un}}{d\Omega}\left[1 + A_\perp cos2\beta + A_y\xi_y + C_y\cos 2\beta\xi_y + \sin 2\beta\cos\delta(C_x\xi_x + C_z\xi_z) + \sin 2\beta\sin\delta(\bar{C}_x\xi_x + \bar{C}_z\xi_z)\right], \quad (41)$$

where $\xi_i, i = x, y, z,$ is the unit vector of the deuteron target vector polarization in its rest system, $A_\perp$ is the asymmetry caused by the linear polarization of the photon beam provided that all other particles are unpolarized (the so-called single beam asymmetry), A_y is the asymmetry due to the vector polarization of the initial deuteron provided the photon beam is unpolarized (the so-called single target asymmetry). The expressions of these asymmetries in terms of the reaction scalar amplitudes are given in the paper [1]. The quantities C_x, C_y, C_z ($\bar{C}_x, \bar{C}_z$) are the spin correlation coefficients caused by the vector polarization of the initial deuteron provided the photon beam is linearly (circularly) polarized. These spin correlation coefficients have the following expressions in terms of the reaction scalar amplitudes

$$\frac{d\sigma_{un}}{d\Omega}C_y = 3\gamma_1 N\,\mathrm{Im}[g_1 g_2^* - a_1 g_5 g_9^* + a_2 g_7 g_8^* - a_3(g_5 g_7^* + g_8 g_9^*)], \\ \frac{d\sigma_{un}}{d\Omega}C_x = 3\gamma_1 N\,\mathrm{Im}[-g_2 g_6^* + a_1 g_3 g_9^* + a_2 g_4 g_7^* + a_3(g_3 g_7^* + g_4 g_9^*)], \quad (42)$$

$$\frac{d\sigma_{un}}{d\Omega}C_z = 3N\,\mathrm{Im}[g_1g_6^* - a_1g_3g_5^* - a_2g_4g_8^* - a_3(g_3g_8^* + g_4g_5^*)],$$

$$\frac{d\sigma_{un}}{d\Omega}\bar{C}_x = -3\gamma_1 N\,\mathrm{Re}[-g_2g_6^* + a_1g_3g_9^* + a_2g_4g_7^* + a_3(g_3g_7^* + g_4g_9^*)],$$

$$\frac{d\sigma_{un}}{d\Omega}\bar{C}_z = -3N\,\mathrm{Re}[g_1g_6^* - a_1g_3g_5^* - a_2g_4g_8^* - a_3(g_3g_8^* + g_4g_5^*)].$$

Note that real amplitudes (such amplitudes arise, for example, when the reaction mechanism is considered in the impulse approximation) lead to zero values of the following spin correlation coefficients C_x, C_y and C_z, i.e., when the photon beam is linearly polarized. In the case of the circularly polarized photon beam, the spin correlation coefficients $\bar{C}_x$ and $\bar{C}_z$ may be non-zero ones.

2. The deuteron target is tensor polarized.

The differential cross section of this reaction for the case of the tensor polarized initial deuteron and elliptically polarized photon beam can be written as

$$\frac{d\sigma}{d\Omega} = \frac{d\sigma_{un}}{d\Omega}\{1 + A_{zz}Q_{zz} + A_{xx}(Q_{xx} - Q_{yy}) + A_{xz}Q_{xz} + \cos 2\beta\left[C_{zz}Q_{zz} + C_{xx}(Q_{xx} - Q_{yy}) + C_{xz}Q_{xz}\right] + \\ + \sin 2\beta\cos\delta(C_{xy}Q_{xy} + C_{yz}Q_{yz}) + \sin 2\beta\sin\delta(\bar{C}_{xy}Q_{xy} + \bar{C}_{yz}Q_{yz})\}, \tag{43}$$

where A_{xx}, A_{xz}, A_{zz} are the asymmetries caused by the tensor polarization of the initial deuteron provided the photon beam is unpolarized and their expressions in terms of the reaction scalar amplitudes are given in the paper [1]. The quantities $C_{xx}, C_{xz}, C_{zz}, C_{xy}, C_{yz}$ ($\bar{C}_{xy}, \bar{C}_{yz}$) are the spin correlation coefficients caused by the tensor polarization of the initial deuteron provided the photon beam is linearly (circularly) polarized. These polarization observables have the following expressions in terms of the reaction scalar amplitudes

$$\frac{d\sigma_{un}}{d\Omega}C_{xx} = \frac{3}{2}N[|g_1|^2 + |g_6|^2 - a_1(|g_3|^2 + |g_5|^2) - a_2(|g_4|^2 + |g_8|^2) - 2a_3\,\mathrm{Re}(g_3g_4^* + g_5g_8^*)], \tag{44}$$

$$\frac{d\sigma_{un}}{d\Omega}C_{xz} = 6N\,\mathrm{Re}[g_1g_2^* - a_1g_5g_9^* - a_2g_7g_8^* - a_3(g_5g_7^* + g_8g_9^*)], \tag{45}$$

$$\frac{d\sigma_{un}}{d\Omega}C_{zz} = 3N\{|g_2|^2 - a_1|g_9|^2 - a_2|g_7|^2 - 2a_3\,\mathrm{Re}(g_7g_9^*) + \tag{46}$$

$$+\frac{1}{2\gamma_1^2}[|g_6|^2 - |g_1|^2 + a_1(|g_5|^2 - |g_3|^2) + a_2(|g_8|^2 - |g_4|^2) + 2a_3\,\mathrm{Re}(g_5g_8^* - g_3g_4^*)]\}, \tag{47}$$

$$\frac{d\sigma_{un}}{d\Omega}C_{xy} = 6N\,\mathrm{Re}[g_1g_6^* + a_1g_3g_5^* + a_2g_4g_8^* + a_3(g_3g_8^* + g_4g_5^*)], \tag{48}$$

$$\frac{d\sigma_{un}}{d\Omega}C_{yz} = 6N\,\mathrm{Re}[g_2g_6^* + a_1g_3g_9^* + a_2g_4g_7^* + a_3(g_3g_7^* + g_4g_9^*)], \tag{49}$$

$$\frac{d\sigma_{un}}{d\Omega}\bar{C}_{xy} = 3N\,\mathrm{Im}[g_1g_6^* + a_1g_3g_5^* + a_2g_4g_8^* + a_3(g_3g_8^* + g_4g_5^*)],$$

$$\frac{d\sigma_{un}}{d\Omega}\bar{C}_{yz} = 3N\,\mathrm{Im}[g_2g_6^* + a_1g_3g_9^* + a_2g_4g_7^* + a_3(g_3g_7^* + g_4g_9^*)]. \tag{50}$$

Note that real amplitudes (the amplitudes in the impulse approximation for this reaction) lead to zero values of the following spin correlation coefficients $\bar{C}_{xy}$ and $\bar{C}_{yz}$, i.e., when the photon beam is circularly polarized. In the case of the linearly polarized photon beam, the rest spin correlation coefficients may be non-zero ones.

Let us calculate the helicity amplitudes for the $\gamma + d \to d + \pi$ reaction.

$$e_\mu^{(\pm 1)} = \frac{1}{\sqrt{2}}(0, \mp 1, -i, 0). \tag{51}$$

The helicity states of the initial deuteron are

$$U_{1\mu}^{(\pm 1)} = \frac{1}{\sqrt{2}}(0, \pm 1, -i, 0), \qquad U_{1\mu}^{(0)} = \frac{1}{M}(-k, 0, 0, E_1), \tag{52}$$

where $k = (W^2 - M^2)/2W$ is the energy of the photon beam in the reaction CMS. Here we take into account the phase agreement for the particle moving along the negative direction of *z* axis. The helicity states of the final deuteron can be written as

$$U_{2\mu}^{(\pm 1)} = \frac{1}{\sqrt{2}}(0, \pm\cos\vartheta, -i, \mp\sin\vartheta), \qquad U_{2\mu}^{(0)} = \frac{1}{M}(-q, E_2\sin\vartheta, 0, E_2\cos\vartheta). \tag{53}$$

We introduce the set of the helicity amplitudes $H_{\lambda_d'}^{\lambda_\gamma \lambda_d}$ (where λ_γ and $\lambda_d(\lambda_d')$ are the helicities of the photon and initial (recoil) deuteron). As it is shown above, the matrix element of this reaction can be described in terms of the scalar amplitudes. The relations between the helicity amplitudes and the scalar amplitudes are the following

$$\begin{gathered}
h_{1,2} = H_{\pm}^{++} = -\frac{ie}{2\sqrt{2}}[g_1 + g_6 \pm \cos\vartheta(g_5 - g_3) \pm \sin\vartheta(g_4 - g_8)], \\
h_{3,4} = H_0^{+\pm} = \frac{ie}{2}\gamma_2[\cos\vartheta(g_4 \mp g_8) + \sin\vartheta(g_3 \mp g_5)], \\
h_{5,6} = H_{\pm}^{+-} = \frac{ie}{2\sqrt{2}}[g_1 - g_6 \pm \cos\vartheta(g_3 + g_5) \mp \sin\vartheta(g_4 + g_8)], \\
h_{7,8} = H_{\pm}^{+0} = -\frac{ie}{2}\gamma_1[g_2 \pm \cos\vartheta g_9 \mp \sin\vartheta g_7], \qquad h_9 = H_0^{+0} = -\frac{ie}{\sqrt{2}}\gamma_1\gamma_2[\cos\vartheta g_7 + \sin\vartheta g_9],
\end{gathered} \tag{54}$$

where $\gamma_2 = E_2 / M$.

Let us present the inverse relations, i.e., the expressions of the reaction scalar amplitudes in terms of the helicity amplitudes. They are

$$\begin{gathered}
g_1 = \frac{i}{\sqrt{2}e}(h_1 + h_2 - h_5 - h_6), \qquad g_2 = \frac{i}{e\gamma_1}(h_7 + h_8), \\
g_3 = -\frac{i}{\sqrt{2}e}[\cos\vartheta(h_1 - h_2 + h_5 - h_6) + \frac{\sqrt{2}}{\gamma_2}\sin\vartheta(h_3 + h_4)], \\
g_4 = -\frac{i}{\sqrt{2}e}[\sin\vartheta(h_2 - h_1 + h_6 - h_5) + \frac{\sqrt{2}}{\gamma_2}\cos\vartheta(h_3 + h_4)], \\
g_5 = -\frac{i}{\sqrt{2}e}[\cos\vartheta(h_2 - h_1 + h_5 - h_6) - \frac{\sqrt{2}}{\gamma_2}\sin\vartheta(h_3 - h_4)], \\
g_6 = \frac{i}{\sqrt{2}e}(h_1 + h_2 + h_5 + h_6), \qquad g_7 = -\frac{i}{e\gamma_1}[\sin\vartheta(h_7 - h_8) - \frac{\sqrt{2}}{\gamma_2}\cos\vartheta h_9], \\
g_8 = -\frac{i}{\sqrt{2}e}[\sin\vartheta(h_1 - h_2 + h_6 - h_5) - \frac{\sqrt{2}}{\gamma_2}\cos\vartheta(h_3 - h_4)], \qquad g_9 = \frac{i}{e\gamma_1}[\cos\vartheta(h_7 - h_8) + \frac{\sqrt{2}}{\gamma_2}\sin\vartheta h_9].
\end{gathered} \tag{55}$$

CONCLUSION

The model-independent analysis of the polarization observables in the process of coherent photoproduction of a pair pseudoscalar mesons on the deuteron target has been done. This analysis does not depend on the details of the reaction mechanism and does not require the knowledge of the deuteron structure. The formalism used in the analysis is based on the most general symmetry properties of the hadron electromagnetic interaction, such as the gauge invariance and P-invariance.

The polarization observables for the process considered are expressed in terms of the reaction scalar amplitudes. The helicity amplitudes describing this reaction are calculated in terms of these amplitudes. The inverse relations, i. e., the expressions of the reaction scalar amplitudes in terms of the helicity amplitudes are also calculated. The features of the reaction $\gamma + d \to d + \pi + \pi$, which are due to the three-body final state, are shortly discussed. The following experimental set-ups have been investigated:

- the photon beam is elliptically polarized (which includes the particular cases of the linear and circular polarizations of the photon beam) and the deuteron target is vector polarized;
- the photon beam is elliptically polarized and the deuteron target is tensor polarized.

The formalism used for the analysis of the polarization observables for the process (1) was applied also to the reaction of coherent photoproduction of pseudoscalar meson on the deuteron target, $\gamma + d \to d + \pi$. We discussed the differences between the polarization observables of this process and of the reaction (1). The helicity amplitudes describing this reaction are calculated in terms of the reaction scalar amplitudes.

Let us note that the new results, which are obtained in this paper, are the following:

- The expressions of the spin correlation coefficients, which are caused by the linear or circular polarization of the photon beam and by the vector or tensor polarization of the deuteron target, in terms of the orthogonal reaction scalar amplitudes.
- The helicity amplitudes in terms of the reaction scalar amplitudes and inverse relations.

They are represented by formulas (4) - (24), (29) - (37) and (B.1) - (B.18).

APPENDIX A

Now we give some formulae describing the polarization state of the deuteron target for different cases. For the case of arbitrary polarization of the target it is described by the general spin–density matrix (in general case it is defined by 8 parameters) which in the coordinate representation has the form

$$\begin{gathered}\rho_{\mu\nu} = -\frac{1}{3}(g_{\mu\nu} - \frac{p_\mu p_\nu}{M^2}) + \frac{i}{2M}\varepsilon_{\mu\nu\lambda\rho}\, s_\lambda\, p_\rho + Q_{\mu\nu}, \\ Q_{\mu\nu} = Q_{\nu\mu}, \qquad Q_{\mu\mu} = 0, \qquad p_\mu Q_{\mu\nu} = 0,\end{gathered} \tag{A.1}$$

where p_μ is the deuteron 4-momentum, s_μ and $Q_{\mu\nu}$ are the deuteron polarization 4-vector and the deuteron quadrupole–polarization tensor.

In the deuteron rest frame the above formula is written as

$$\rho_{ij} = \frac{1}{3}\delta_{ij} - \frac{i}{2}\varepsilon_{ijk}s_k + Q_{ij}, ij = x, y, z. \tag{A.2}$$

This spin–density matrix can be written in the helicity representation using the following relation

$$\rho_{\lambda\lambda'} = \rho_{ij} e_i^{(\lambda)*} e_j^{(\lambda')}, \qquad \lambda, \lambda' = +, -, 0, \tag{A.3}$$

where $e_i^{(\lambda)}$ are the deuteron spin functions which have the deuteron spin projection λ on to the quantization axis (z axis). They are

$$e^{(\pm)} = \mp\frac{1}{\sqrt{2}}(1, \pm i, 0), \qquad e^{(0)} = (0, 0, 1). \tag{A.4}$$

The elements of the spin–density matrix in the helicity representation are related to the ones in the coordinate representation by such a way

$$\begin{gathered}\rho_{\pm\pm} = \frac{1}{3} \pm \frac{1}{2}s_z - \frac{1}{2}Q_{zz}, \qquad \rho_{00} = \frac{1}{3} + Q_{zz}, \qquad \rho_{+-} = -\frac{1}{2}(Q_{xx} - Q_{yy}) + iQ_{xy}, \\ \rho_{+0} = \frac{1}{2\sqrt{2}}(s_x - is_y) - \frac{1}{\sqrt{2}}(Q_{xz} - iQ_{yz}), \qquad \rho_{-0} = \frac{1}{2\sqrt{2}}(s_x + is_y) + \frac{1}{\sqrt{2}}(Q_{xz} + iQ_{yz}), \qquad \rho_{\lambda\lambda'} = (\rho_{\lambda'\lambda})^*.\end{gathered} \tag{A.5}$$

To obtain these relations we use $Q_{xx} + Q_{yy} + Q_{zz} = 0$.

The polarized deuteron target which is described by the population numbers n_+, n_- and n_0 is often used in the spin experiments (see, for example, Ref. [7]). Here n_+, n_- and n_0 are the fractions of the atoms with the nuclear spin projection on to the quantization axis $m=+1$, $m=-1$ and $m=0$, respectively. If the spin–density matrix is normalizd to 1, i.e. $Sp\rho=1$, then we have $n_+ + n_- + n_0 = 1$. Thus, the polarization state of the deuteron target is defined in this case by two parameters: the so–called *V* (vector) and *T* (tensor) polarizations

$$V = n_+ - n_-, \qquad T = 1 - 3n_0. \tag{A.6}$$

Using the definitions for the quantities $n_{\pm,0}$

$$n_\pm = \rho_{ij} e_i^{(\pm)*} e_j^{(\pm)}, \qquad n_0 = \rho_{ij} e_i^{(0)*} e_j^{(0)}, \tag{A.7}$$

we have the following relation between *V* and *T* parameters and parameters of the spin–density matrix in the coordinate representation (in the case when the quantization axis is directed along the *z* axis)

$$n_0 = \frac{1}{3} + Q_{zz}, \qquad n_\pm = \frac{1}{3} \pm \frac{1}{2} s_z - \frac{1}{2} Q_{zz}, \tag{A.8}$$

or

$$T = -3Q_{zz}, \qquad V = s_z. \tag{A.9}$$

APPENDIX B

In this Appendix we present the expressions for the reaction scalar amplitudes $f_i (i = 1-9)$ and $g_i (i = 10-18)$ in terms of the helicity amplitudes $H_i (i = 1-18)$. These expressions are

$$\begin{aligned}
f_1 &= \frac{1}{2\sqrt{2}e}[i\sin\varphi\,(H_2 - H_1 + H_{17} - H_{18}) - (\cos\theta\cos\varphi + i\sin\varphi)\,(H_4 - H_3 + H_7 - H_8) + \\
&\quad + \frac{\sqrt{2}}{\gamma_2}\sin\theta\cos\varphi\,(H_{14} - H_{13} + H_{15} - H_{16})], \\
f_2 &= \frac{1}{2\sqrt{2}e}[(\cos\varphi + i\cos\theta\sin\varphi)\,(H_4 - H_3 + H_8 - H_7) - (\cos\varphi - i\cos\theta\sin\varphi)\,(H_1 - H_2 + H_{17} - H_{18}) - \\
&\quad - i\frac{\sqrt{2}}{\gamma_2}\sin\theta\sin\varphi\,(H_{13} - H_{14} + H_{15} - H_{16})], \\
f_3 &= \frac{1}{2e\gamma_1}[\sin\theta\,(H_{10} - H_9 + H_{11} - H_{12}) + \frac{\sqrt{2}}{\gamma_2}\cos\theta\,(H_6 - H_5)], \\
f_4 &= \frac{1}{2\sqrt{2}e}[\sin\theta\,(H_2 - H_1 + H_4 - H_3 + H_7 - H_8 + H_{17} - H_{18}) + \frac{\sqrt{2}}{\gamma_2}\cos\theta\,(H_{14} - H_{13} + H_{15} - H_{16})], \\
f_5 &= \frac{1}{2e\gamma_1}[(\cos\theta\cos\varphi + i\sin\varphi)\,(H_{12} - H_{11}) + (\cos\theta\cos\varphi - i\sin\varphi)\,(H_9 - H_{10}) + \frac{\sqrt{2}}{\gamma_2}\sin\theta\cos\varphi\,(H_6 - H_5)], \\
f_6 &= \frac{1}{2\sqrt{2}e}[(\cos\varphi - i\cos\theta\sin\varphi)\,(H_1 + H_2 - H_{17} - H_{18}) - (\cos\varphi + i\cos\theta\sin\varphi)\,(H_3 + H_4 - H_7 - H_8) + \\
&\quad + i\frac{\sqrt{2}}{\gamma_2}\sin\theta\sin\varphi\,(H_{13} + H_{14} - H_{15} - H_{16})], \\
f_7 &= \frac{1}{2e\gamma_1}[(\cos\varphi - i\cos\theta\sin\varphi)\,(H_9 + H_{10}) + (\cos\varphi + i\cos\theta\sin\varphi)\,(H_{11} + H_{12}) + i\frac{\sqrt{2}}{\gamma_2}\sin\theta\sin\varphi\,(H_5 + H_6)], \\
f_8 &= -\frac{i}{2\sqrt{2}e}[(\sin\varphi + i\cos\theta\cos\varphi)\,(H_1 + H_2 + H_{17} + H_{18}) + (\sin\varphi - i\cos\theta\cos\varphi)\,(H_3 + H_4 + H_7 + H_8) - \\
&\quad - i\frac{\sqrt{2}}{\gamma_2}\sin\theta\cos\varphi\,(H_{13} + H_{14} + H_{15} + H_{16})], \\
f_9 &= \frac{1}{2\sqrt{2}e}[\sin\theta\,(H_3 + H_4 + H_7 + H_8 - H_1 - H_2 - H_{17} - H_{18}) - \frac{\sqrt{2}}{\gamma_2}\cos\theta\,(H_{13} + H_{14} + H_{15} + H_{16})],
\end{aligned} \tag{B.1}$$

$$g_{10}=\frac{i}{2\sqrt{2}e}[(\cos\varphi-i\cos\theta\sin\varphi)(H_1-H_2+H_{18}-H_{17})+(\cos\varphi+i\cos\theta\sin\varphi)(H_4-H_3+H_7-H_8)-$$
$$-i\frac{\sqrt{2}}{\gamma_2}\sin\theta\sin\varphi(H_{14}-H_{13}+H_{15}-H_{16})],$$

$$g_{11}=-\frac{i}{2e\gamma_1}[(\cos\varphi-i\cos\theta\sin\varphi)(H_{10}-H_9)+(\cos\varphi+i\cos\theta\sin\varphi)(H_{12}-H_{11})+i\frac{\sqrt{2}}{\gamma_2}\sin\theta\sin\varphi(H_6-H_5)],$$

$$g_{12}=\frac{1}{2\sqrt{2}e}[(i\cos\theta\cos\varphi-\sin\varphi)(H_4-H_3+H_8-H_7)+(\sin\varphi+i\cos\theta\cos\varphi)(H_1-H_2+H_{17}-H_{18})-$$
$$-i\frac{\sqrt{2}}{\gamma_2}\sin\theta\cos\varphi(H_{13}-H_{14}+H_{15}-H_{16})],$$

$$g_{13}=-\frac{i}{2\sqrt{2}e}[\sin\theta(H_1-H_2+H_4-H_3+H_8-H_7+H_{17}-H_{18})+\frac{\sqrt{2}}{\gamma_2}\cos\theta(H_{13}-H_{14}+H_{15}-H_{16})],$$

$$g_{14}=\frac{1}{2\sqrt{2}e}[(\sin\varphi-i\cos\theta\cos\varphi)(H_3+H_4-H_7-H_8)-(\sin\varphi+i\cos\theta\cos\varphi)(H_1+H_2-H_{17}-H_{18})+$$
$$+i\frac{\sqrt{2}}{\gamma_2}\sin\theta\cos\varphi(H_{13}+H_{14}-H_{15}-H_{16})],$$

$$g_{15}=\frac{i}{2\sqrt{2}e}[(\cos\varphi-i\cos\theta\sin\varphi)(H_1+H_2+H_{17}+H_{18})+(\cos\varphi+i\cos\theta\sin\varphi)(H_3+H_4+H_7+H_8)+$$
$$+i\frac{\sqrt{2}}{\gamma_2}\sin\theta\sin\varphi(H_{13}+H_{14}+H_{15}+H_{16})],$$

$$g_{16}=\frac{i}{2e\gamma_1}[\sin\theta(H_9+H_{10}-H_{11}-H_{12})+\frac{\sqrt{2}}{\gamma_2}\cos\theta(H_5+H_6)],$$

$$g_{17}=\frac{i}{2\sqrt{2}e}[\sin\theta(H_1+H_2+H_3+H_4-H_7-H_8-H_{17}-H_{18})+\frac{\sqrt{2}}{\gamma_2}\cos\theta(H_{13}+H_{14}-H_{15}-H_{16})],$$

$$g_{18}=\frac{1}{2e\gamma_1}[-(\sin\varphi+i\cos\theta\cos\varphi)(H_9+H_{10})+(-\sin\varphi+i\cos\theta\cos\varphi)(H_{11}+H_{12})+i\frac{\sqrt{2}}{\gamma_2}\sin\theta\cos\varphi(H_5+H_6)],$$

where e is the electron charge, θ and φ are the polar and azimuthal angles of the recoil deuteron momentum, $\gamma_1=E_i/M$, $i=1,2$ and $E_1(E_2)$ is the energy of the initial (final) deuteron in the $\gamma+d\to d+\pi+\pi$ reaction CMS, M is the deuteron mass.

REFERENCES

1. Gakh G.I., Rekalo A.P. and Gakh A.G. General analysis of polarization effects in the reaction $\gamma+d\to d+\pi+\pi$. I. Single-spin asymmetries //The Journal of Kharkiv National University, physical series Nuclei, Particles, Fields. - 2011. - №969. - Issue 3 /51/. - P. 4-15.
2. Krusche B. and Schadmand S. Study of non-strange baryon resonances with meson photoproduction // Prog. Part. Nucl. Phys. - 2003. - Vol. 51. - P. 399-485.
3. MacCormick M. et al. Total photoabsorption cross sections for ^{1}H, ^{2}H, and ^{3}He from 200 to 800 MeV // Phys. Rev. - 1996. - Vol. C53. - P. 41-49.
4. Krusche B. Meson photoproduction in the first and second resonance region // arXiv:nucl-ex/0304008.
5. Crede V. Light Baryon Spectroscopy using the CLAS Spectrometer at Jefferson Laboratory // arXiv:1108. 5199v1 [nucl-ex] 2011.
6. Gibson W.M. and Pollard B.R. Symmetry principles in elementary particle physics. – Cambridge – London – New York – Melbourne: Cambridge University Press, 1976. – 343 p.
7. Airapetain A. et al. Measurement of the Tensor Structure Function b_1 of the deuteron // Phys. Rev. Lett. - 2005. - Vol. 95. - P. 242001. (6 pgs.)