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One-dimensional modulational instability models of intense Langmuir plasma oscillations using the Silin–Zakharov equations

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Contents

Abstract. **The modulational instability mechanisms of intense Langmuir oscillations in a plasma are reviewed both for field energy densities below (Zakharov's model) and above (Silin's model) the plasma's thermal energy density. It is shown by a one-dimensional example that V E Zakharov's mechanism involving nonlinear absorption of Langmuir oscillations in plasma also holds for intense cold plasma fields described by V P Silin's model. It is also shown that the development mechanisms of the modulational instability of Langmuir oscillations are similar for nonisothermal and cold plasmas. Hybrid models treating electrons quasihydrodynamically and ions as particles are analyzed in detail, which allows the study of the direct mechanism by which energy is transferred to ions in the instability development process.**

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1. Introduction

Intense Langmuir waves in plasma, which are easily excited by different sources [1–9], turn out to be parametrically unstable. This instability is responsible for the excitation of a short-wavelength oscillation spectrum synchronized in frequency to the intense Langmuir (pump) wave and for the formation of deep plasma density caverns filled with a high-frequency (HF) field. Interest in these processes was due, in particular, to the opening up of the feasibility of electron and ion heating. The correct tools for describing the parametric instability of long-wavelength Langmuir oscillations were practically developed in the basic work by V P Silin [10] and V E Zakharov [11]. Even the first one-dimensional numerical experiments on the parametric decay of Langmuir oscillations [12] bore out these theoretical predictions [7] (see also Refs [13, 14] and review [15]). The complete theory of parametric plasma oscillation decay was presented in monograph [16] more recently.

However, the pronounced interest of the scientific community was aroused by the effective mechanism of wave energy dissipation discovered and clarified by V E Zakharov — the collapse of Langmuir waves in nonisothermal plasmas [17]. This is the formation of the short-wavelength perturbation spectrum and plasma density caverns, which may be described by Zakharov's equations [17], which were derived with the use of hydrodynamic equations for electron and ion fluids assuming that the energy density of the long-wavelength Langmuir field was lower than the thermal energy density of plasma electrons. In Zakharov's hydrodynamic model, localization domains of short-wavelength Langmuir oscillations emerge. The plasma is forced out of these domains (caverns) under HF radiation pressure, so that the plasma density turns out to be appreciably lower than the volume average density. The subsequent evolution may lead to a so-called collapse — the shrinkage and deepening of the density cavern (the so-called peaking mode). In this case, the cavern shrinkage, as may be seen from more general models describing this phenomenon, should be attended by the electron damping of small-scale HF spectrum modes and the cavern 'collapse' due to HF field burnup (so-called 'physical collapse').

Even early in the study of these processes, analytical investigations as well as hardware and numerical experiments bore out [18–21] the fact that an appreciable energy fraction of intense Langmuir oscillations in a nonisothermal plasma is converted to the energy of the short-wavelength Langmuir spectrum due to modulation instability. A like effect was also discovered in stronger fields in cool plasmas [22, 23], where the field–particle energy transfer mechanism turned out to be similar. This signifies that the nonlinear mechanism of Langmuir oscillation absorption operative when the thermal plasma energy density exceeds the energy density of the HF field, which was discovered by V E Zakharov [17], turned out to be applicable also to fields whose energy density is far greater than the thermal plasma energy. Subsequently, there followed a wealth of papers dedicated to this phenomenon, which is of utmost importance to plasma physics (see, for instance, Refs [24–34]). Special mention should be made of a paper by E A Kuznetsov [35], who most correctly derived Zakharov's model equations describing the modulation instability of Langmuir waves in nonisothermal plasmas. The reader is referred to reviews [36, 37], which give an idea of the scale and efficiency of this research.

The phenomenon of wave energy absorption due to the development of small-scale modulation instability discovered by V E Zakharov has been elaborated in several applications. Many models describing these processes differ from the previous traditional ones, more and more new features are revealed, and new implications of modulation instability development are highlighted.

It is clear that there is no way of including several kinetic effects (for instance, Landau damping) in a hydrodynamic model. That is why use is commonly made of a phenomenological description of this phenomenon by introducing the corresponding terms into the system of hydrodynamic equations. This is admissible to a certain degree, because the nature of Landau damping has been adequately studied. On the other hand, the behavior of particles trapped by a spatially nonuniform field is not quite correctly described by a purely hydrodynamic treatment: their inertia (significant precisely for ions) is in fact ignored. This gives rise not only to deep plasma density caverns of a very small scale, but also to peaking modes, which are not always adequate for the physical reality.

To correctly include Landau damping by electrons, advantage is frequently taken of the kinetic equation for their distribution function. However, it is good to bear in mind that the kinetic damping due to electrons can, under certain conditions, disturb the conditions for modulation instability development by suppressing the field even at the stage of forming caverns which may be distorted in shape in the process. Therefore, there are problems in the interpretation of the process of modulation instability, whose character may markedly change on engaging strong kinetic damping. Furthermore, the kinetic approach, like the hydrodynamic one, describes the motion of a continuous medium and allows the existence of physically unpromising solutions with peakings reaching arbitrarily small scales.

Below, we discuss different models for describing the modulation instability of intense Langmuir oscillations in plasma in a one-dimensional representation. As noted by J M Dawson [38], the choice of one-dimensional models retains the main features of the processes, while significantly simplifying the description and understanding of the physical phenomena. Furthermore, the main difficulty in describing plasma in three-dimensional models is not only the difference in electron and ion masses, but also a very large number of particles (electrons and ions) — on the order of $10^{12} - 10^{15}$ or more per unit volume — compeling the construction of rather intricate models, which nevertheless still remain approximate. This hinders the comparison between hydrodynamic, kinetic models and models that use, partly or fully, descriptions with the aid of large particles, particle-in-cell (PIC) method, etc., simulating the behavior of ions and electrons.

In the one-dimensional models corresponding to the three-dimensional case given above, the number of particles corresponding to ions and electrons is on the order of $10^4 - 10^5$ per unit volume, and these particles are therefore close in characteristics to plasma ions and electrons. Therefore, a description involving particles may turn out to be more correct in the framework of a one-dimensional model than a hydrodynamic description or a description based on the kinetic equations for their distribution function. This may permit elucidating the question of the appropriateness of different ways of process description.

1.1 Nonisothermal plasma

The greatest progress was achieved in the study of the modulation instability of an intense Langmuir field in nonisothermal plasmas for a field energy density well below the electron thermal energy density.

In the one-dimensional case, in a nonisothermal plasma a small-scale soliton-like cavern forms, where the HF radiation pressure is balanced by the plasma electron pressure (see, for instance, Ref. [39]). However, it is possible to observe the 'collapse' of plasma density caverns in these low-dimensional cases, too, when the HF pressure lowers due to field burnup caused by Landau damping [40]. Broadly speaking, the cavern collapse maintains the heating of not only electrons but also ions; it increases the entire plasma pressure, which also disturbs the equilibrium state of these structures. In a supersonic mode of cavern wall motion, the probability of a physical collapse may rise even in the one-dimensional case. The modulation instability of an intense Langmuir wave in a nonisothermal plasma has also resulted in collective ion excitations, and in the generation of ion-acoustic waves, in particular [41–44].

A comparison between the one-dimensional Vlasov–Poisson kinetic model, which describes the behavior of electrons and ions with the aid of kinetic equations for the distribution functions, and Zakharov's hydrodynamic model at the same parameter values and the same initial conditions was undertaken, for instance, in Ref. [45], where the amplitude of the long-wavelength field (the pump) did not vary with time. The most adequate was the comparison for the nonisothermal plasma case. In the cavern formation early in the nonlinear process in the constant pump mode, one can see differences in the formation of density caverns whose shape in the kinetic model does not correspond to the perturbation structure typical for the modulation instability. Although in both cases the HF field-induced plasma expulsion gives rise to lower-density domains, the magnitude of density changes in Zakharov's model turned out to be significantly larger than in the Vlasov–Poisson model. Therefore, it was shown that kinetic field damping on plasma particles can distort the modulation instability process and, perhaps, lead to other consequences, in particular, giving rise to groups of fast particles and early disruption of density caverns.

It is highly instructive to compare Zakharov's hydrodynamic model with the model which describes electrons using the kinetic equations for their distribution function and treats ions hydrodynamically [46]. The case of constant pumping was also considered here. This model describes much better the formation of caverns typical for a developed modulation instability, which are hardly different, early in the nonlinear process, from the structures of this kind in Zakharov's hydrodynamic model. A remark is in order regarding the models which apply this kinetic description of the electron plasma component and the hydrodynamic approach to the ion component: not only do they permit describing the formation of plasma density caverns, but they are also able to determine more precisely the characteristics of the electron velocity distribution, in particular, the electron temperature, although they remain unable to provide an answer to questions regarding the ion energy distribution.

In the representation of ions by particles in the framework of the so-called hybrid models[1] (the electrons are described hydrodynamically, and the ions are treated as large particles), ion density fluctuations prove to be quite significant [47–49], at least in the one-dimensional Zakharov's nonisothermal plasma models under discussion. This speeds up the development of modulation instability to the extent that the linear stage of perturbation growth practically escapes observation (although this, as noted below, is due to the fact that the instability increment turns out to be almost the same throughout a wide range of wavenumber values for the supersonic modes of the process under discussion).

A treatment in the framework of such hybrid models would permit taking into account the inertia of ions in the formation and evolution of plasma density caverns, in particular, the mechanism of cavern collapse. It is precisely the direct simulation of the collapse by the particle method that is 'most consistent', in the view of V E Zakharov and his colleagues expressed in Ref. [50]. Indeed, the kinetic and hydrodynamic descriptions operate on objects that are small phase volumes rather than particles, and these phase volumes become arbitrarily small when passing to the classical limit. This leads to a smaller inertia of the substance than in its description by particles.

[1] This name was proposed by Clark et al. [49].

As for the description methods with the aid of large particles in high-dimensional models, this is another extreme. Large particles possess excessive inertia and, therefore, they are quite often replaced with local objects — computation cells — in which the inner contents are averaged. This brings such an approach closer to the hydrodynamic scale description, retaining the features of the large-particle technique and their averaged inertia on a long scale. It is possible to increase the number of model particles in the description, decreasing the fraction (charge and mass) in each of them, although it is hardly possible to approach the real physical parameters in the three-dimensional (3D) space.

In what follows, the emphasis is placed on one-dimensional hybrid models. For one-dimensional simulations, we employ $(2-5)\times10^4$ model ion-particles (which would correspond to $10^{13}-10^{14}$ such objects in the volume under consideration in the 3D model), with the characteristics of these particles already corresponding to single ions. That is why the dynamics of ion-simulating particles in this case is largely adequate to the dynamics of plasma ions; furthermore, the particle–field energy exchange mechanisms correspond to the real interaction of ions with the low-frequency (LF) oscillation spectrum. This signifies that one-dimensional hybrid models with a large number of particles are able to provide a correct description of the nonlinear Landau damping of slow plasma density perturbations on the ions, leaving beyond the scope of this approach the problems of describing the details of the electron distribution function. The inclusion of the nonresonance interaction of ion-particles with LF spectrum modes and the capture of ions in the potential wells of such oscillations result in an additional instability of density caverns arising due to modulation instability, as well as in the emergence of fast particle groups.

The authors of Ref. [49] undertook a comparison between two models — the hydrodynamic and hybrid Zakharov's — at the same parameter values and the same initial conditions. Because of a higher level of ion density fluctuations, the number of caverns in the hybrid model turned out to be appreciably larger, and they were less deep than in Zakharov's model. The integral characteristics of both models proved to be practically the same. A drawback of the work performed by these authors is a nonself-consistent description, i.e., neglect of the effect of the spectrum under excitation on the pump wave. It should be emphasized that in the cases of description based on the hydrodynamic Zakharov's model [49] and of the description in the framework of kinetic equations for the electron distribution function and hydrodynamic treatment for ions [51], the caverns remained immobile, which was not observed in the hybrid model.

1.2 Cold plasma

With the advent of high-power energy sources, which excited highly intense Langmuir oscillations whose field energy density was far greater than the electron thermal energy density, the model developed by V P Silin [10, 16] and further elaborated by him and his coworkers when describing the parametric instability of an intense field in a cold plasma came into demand. Under these conditions, the dispersion term in the equation for the Langmuir wave field caused by thermal plasma motion is rather small and, assuming the plasma to be cold, may be ignored in many cases.

Indeed, when the field energy density is appreciably higher than the plasma thermal energy density, modulation instability develops, at least early in the process, according to the scenario proposed by Silin et al. [10, 22], in whose models a powerful Langmuir wave in a cold plasma induces intense electron velocity oscillations whose amplitude is comparable to the wavelength of the modes of the spectrum under excitation. In this case, generally speaking, the instability should be termed parametric [16]. Both Zakharov's and Silin's models nevertheless turn out to be physically similar [52]. This is precisely why the term 'modulation instability' applies to the description of the instability of the powerful Langmuir field in Silin's model as well.

Notably, even in one-dimensional numerical simulations of the process, proceeding from the hydrodynamic Silin equations generalized in Refs [53, 54], the modulation instability developed and a partial energy exchange occurred between its short-wavelength spectrum and an intense pump wave. The results of such simulations are in qualitative and quantitative agreement with the results of numerical experiments performed earlier at the P N Lebedev Physical Institute [22]. A peaking mode, which was characterized by a shortening of the cavern scale length and simulation breakdown, could be observed. It is the latter circumstance that compelled us to move to a description of ions as particles.

In the hybrid Silin model (electrons are hydrodynamically described, and ions are treated as large particles), in precisely the same way density caverns formed, which then collapsed [48]. This was caused not only by the nonequilibrium initial state of the caverns (due to violation of the balance between the HF pressure and the plasma pressure) and the field burnup effect, but also by the inclusion of inertia of ion-simulating particles whose number was not large enough in the numerical experiments. In this case, the ion cavern 'collapsed' and the ion component passed into the particle trajectory crossing mode [47, 48]. The energy extracted by the ions was on the order of $(m_e/M)^{1/3}$ fraction of the initial energy of the pump wave [48] (here, m_e and M are the electron and ion masses, respectively). For electrons, the passage to the trajectory crossing mode could be restrained by the existence of the ion cavern, which was capable of synchronizing the ejection of fast electrons and ions at the instant of its collapse. Experiments were made to produce — in the vicinity of the plasma resonance in a nonuniform plasma — a close-to-Langmuir-frequency field with a high energy density W exceeding the plasma thermal energy density $n_{e0}T_{e0}$. These experiments demonstrated the generation of short fast-particle pulses against the background of electron heating in the vicinity of the plasma resonance. In this case, the energy was removed from the domain of plasma resonance not only by electrons, but also by ions [55–57] of rather high energy (see, for instance, review [57]). The domains of electron pulse sources corresponded to the small dimensions of the plasma density caverns. The energy fraction stored in the fast ions after the cavern collapse was roughly consistent with the theoretical values given in Refs [48, 58–60].

In the literature, the instability of oscillatory electron motion at the Langmuir frequency relative to immobile ions was quite often referred to as the oscillatory Buneman instability. The similarity between the Buneman instability and the Langmuir wave instability in a cold plasma is attested to by the fact that the increments and the initial velocities of

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the relative motion of electrons and ions are approximately equal. An analysis of Buneman instability development was outlined in monograph [6], where a lowering of the velocity (current disruption) of the relative motion and a growth of perturbations of the electron and ion components were observed at the nonlinear stage of the process. This is in qualitative correspondence with the processes occurring in the development of the parametric instability of an intense Langmuir wave in a cold plasma.

The parametric instability of Langmuir waves under the applicability conditions of Zakharov's equations and Silin's equations was usually discussed by theorists separately, although quite often these processes were not distinguished in experiments. It would therefore be instructive to compare the behavior of the parametric instability of intense Langmuir oscillations in hot and cold plasmas in the framework of hybrid self-consistent models. The bulk of attention was directed towards the behavior of the ion plasma component. It turned out that the HF field energy fraction transferred to ions in the nonisothermal plasma was on the order of W/n_0T_{e0}, while in the cold plasma case an estimate [48] on the order of $(m_e/M)^{1/3}$ was confirmed. In the latter case, the fraction of fast particles in their energy distribution turned out to be larger [58–60].

In our paper, special emphasis is also placed on a comparison of the character of exciting the collective degrees of freedom in low-frequency motions, particularly, of the generation of ion waves in the hybrid Zakharov and Silin models. It is also important to elucidate how the rate of HF field burnup in plasma caverns affects the character of ion dynamics. These and other questions are discussed below.

1.3 Comparison between Zakharov's and Silin's models

The main objective of this paper is to discuss different one-dimensional models in order to describe the modulation instability of intense long-wavelength Langmuir oscillations and to elucidate the features of energy transfer to ions and collective ion perturbations in nonisothermal and cold plasmas [59–61].

As shown below, the description of the parametric instability of the intense long-wavelength Langmuir field in plasmas with the excitation of a short-wavelength Langmuir oscillation spectrum is universal both for a cold plasma (i.e., when the field energy density exceeds the thermal energy density of the medium, $W = |E_0|^2/4\pi \gg n_0T_e$) and for a nonisothermal plasma (when the plasma thermal energy density exceeds the field energy density, $W = |E_0|^2/4\pi \ll n_0T_e$, where E_0 is the initial strength of the long-wavelength Langmuir wave field, n_0 is the unperturbed plasma density, T_e is the electron temperature, and the ions are assumed to be cold). To obtain the systems of equations for each of Silin's and Zakharov's models, we therefore take advantage of the approach outlined in V P Silin's book [16].

Although the Silin and Zakharov models under discussion were intended for different physical conditions, constructed relatively long ago, and developed independently for a long time, there has so far been no clear understanding of the close relationship between them. In this study, we endeavored to demonstrate this relationship and highlight the similarity of the physical mechanisms underlying the phenomena described by these models, which is important, in particular, from the methodological standpoint.

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