

A mixture model of differentially growing biological continua

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Continual and discrete-continual models are widely used for biological growth description. Single-cell-based discrete-continual models and uniform continual models describe the growing continua as viscoelastic media with distributed sources of mass, momentum and energy. The growing body is considered as an open thermodynamic system and the mass and energy exchange with environment is provided by the conducting systems (blood vessels and airways in animals, water and sugar conducting systems in plants, transportation systems for the trophic fluids in sponges and molluscs, etc) which are considered as a part of the growing body. Recently a number of mixture models for the growing biological tissues have been proposed. The growing skeleton can be considered as a solid phase and a few liquid phases including the intracellular, extracellular and delivering liquids provide the mass exchange describing the growth as a liquid/solid phase transition.

Numerous experiments and observations of the growing bodies revealed some regularities including the linear dependence between the strain rate and stress tensors and existence of the own growth strain rate, which is nonzero at a zero external load. The growth can be stimulated/oppressed by stretching/compression in the corresponding direction. Commonly the compression leads to the growth oppression in the same direction and the compensative growth stimulation in the orthogonal direction. Since the collagen tissues (bones, cartilage, skin, vessel walls) possess the piezoelectric properties, the growth stimulation/oppression is connected with mechanic-electric coupling and control over the cell growth activity by electric signals. Local growth in plant and animal tissues is characterized by fibre elongation according to the principals of the stress tensor. In some cases the growing surfaces become folded and the possible role of instability of the growing fronts is discussed. The conducting elements in the growing body form a structure with optimal properties for the fluid delivery and mechanical strength. A successive mathematical model must describe and explain the observed growth regularities clarifying the role of mechanical forces in the biological growth and predicting the shape and inner structure of the growing materials for the tissue engineering purposes.

In the paper the differential non uniform biological growth is considered. The conducting elements (blood vessels, airways, conducting elements in plants) and the main tissue (parenchyma) grow with different growth rates. For instance, plant leaves begin their growth as bunches of cylinders forming the main veins of the future leaf blade. The growth of veins is based on cell elongation rather than cell divisions, while the growth of the main tissue is determined by divisions. The two growth rates are coordinated at common interfaces formed by the solid cell walls. Stretching produced by elongating veins is transferred to the main tissue stimulating the divisions of the parenchyma cells. Another example is a mixture of the normal and malignant cells growing at different rates and the boundary conditions allow slipping at the interfaces. In the differential growth models the relative displacement of different solid phases is determined by the interphase forces that must be determined. The proposed model is composed by several continua with different growth rates, viscoelastic properties and boundary conditions at interfaces. Some model problems are solved. The conditions of synchronous growth are obtained. It was found in some cases when the boundary conditions allow the synchronous growth it may be unstable. The instability leads to divergence of the cells of one of the population and pattern formation in the growing mixture.