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Scaling Law for a Low-Pressure Gas Breakdown in a Homogeneous DC Electric Field

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Gas breakdown in nitrogen, air, and oxygen in a dc electric field at various interelectrode distances L is studied experimentally. A scaling law for a low-pressure gas breakdown $U_{dc} = f(pL, L/R)$ is deduced. According to this scaling law, the breakdown voltage U_{dc} is a function not only of the product of the gas pressure p and the gap length L, but also of the ratio of the gap length L to the chamber radius R. It is shown that, for any dimensions of the cylindrical discharge chamber (in the range of L/R under investigation), the ratio of the breakdown electric field to the gas pressure p at the minimum of the ignition curve remains constant: $(E_{dc}/p)_{\min} \approx$ const. A method for calculating the ignition curve in a cylindrical discharge chamber with arbitrary values of L and R is proposed. *© 2000 MAIK "Nauka/Interperiodica".*

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As is known [1–7], the ignition curves of a glow discharge are described by the Paschen law $U_{dc} = f(pL)$; i.e., the breakdown voltage U_{dc} is a function of the product of the gas pressure p and the interelectrode distance L. The Paschen law implies that the ignition curves $U_{dc}(p)$ measured for various distances L must coincide if they are drawn as the function $U_{dc}(pL)$. However, the measurements of the ignition curves of a glow discharge in neon [8] showed that, with equal values of the product pL, the breakdown voltage for a long discharge gap with planar electrodes is significantly higher than that for a short gap. More recent studies [9–14] confirmed this conclusion for some other gases (neon, argon, nitrogen, hydrogen, etc.). In spite of a great number of experimental and theoretical papers devoted to low-pressure gas breakdown in a dc electric field, a method for calculating the ignition curve at arbitrary values of the interelectrode distance L and the radius of the discharge chamber R is still lacking.

This paper is devoted to the experimental study of a breakdown in nitrogen, air, and oxygen in a dc electric field in a discharge chamber with a variable interelectrode distance L. It is shown that, in the range of the ratio L/R under study, the ignition curves shift toward high values of the product pL and discharge voltage U_{dc} as the gap length L increases. In this case, for any values of the gap length L, the ratio of the breakdown electric field to the gas pressure $(E_{dc}/p)_{\min}$ at the minimum of the ignition curve remains constant. A generalized scaling law for the low-pressure gas breakdown $U_{dc} = f(pL, L/R)$ is deduced. A method allowing one to calculate the ignition curve for a glow discharge in a cylindrical chamber with arbitrary dimensions from the known ignition curve for a narrow discharge gap (for $L/R \longrightarrow 0$), i.e., from the usual Paschen curve, is described.

We measured the ignition curves for a glow discharge in the range of dc voltages $U_{dc} \leq 1000$ V and pressures of $p \approx 10^{-2}$–10 torr. A discharge tube with an inner diameter of 63 mm was used. The interelectrode distance L was varied in the range 0.5–10 cm; consequently, the studies were conducted in the range L/R = 0.16–3.2. Planar parallel electrodes spanned the entire cross section of the discharge tube. Both the anode and the cathode were made from stainless steel. The breakdown voltage was measured accurate to ±2 V. When determining the ignition voltage, the growth rate of the discharge voltage did not exceed 1 V/s. In all cases, our procedure for measuring the ignition curves was as follows. We fixed a certain distance L between the electrodes and then, for various gas pressures p, measured the breakdown voltage U_{dc}. Below, we explain why only this way of measuring the ignition curves of a glow discharge is correct.

Figure 1 shows the ignition curves measured by us in nitrogen for different distances L between the electrodes. It is seen from the figure that, as L increases, the ignition curves shift not only toward higher ignition voltages U_{dc} (as was obtained in [9–14]), but also toward higher values of pL. Apparently, such a shift of the ignition curves toward higher values of U_{dc} and pL with increasing interelectrode distance L may be attributed to an increase in losses of charged particles on the

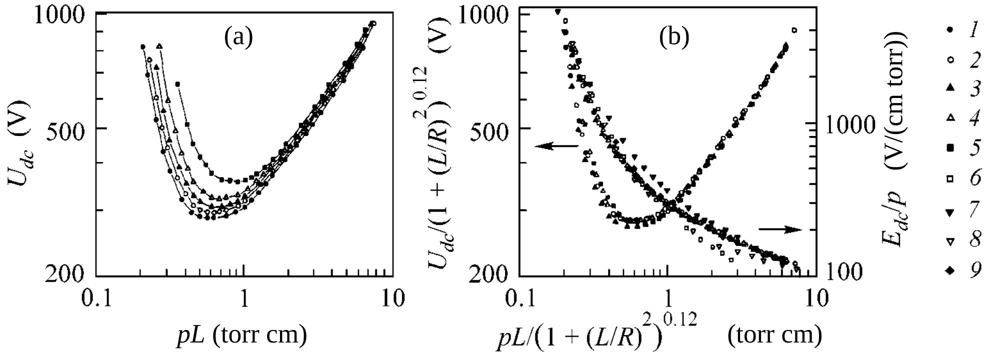

Fig. 1. (a) Experimental ignition curves of a glow discharge in nitrogen for the interelectrode distances L = (*1*) 0.5, (*2*) 2, (*3*) 4, (*4*) 6, and (*5*) 8 cm. (b) Dependences of U^*_{dc} and E_{dc}/p on $(pL)^*$ obtained from curves (*1*)–(*5*) in Fig. 1a and the data from (*6*) [11], (*7*) [15], (*8*) [16], and (*9*) [18].

cylindrical wall of the discharge tube due to diffusion across the electric field.

Figure 2 shows the dependences of $U_{\min}$ and $(E_{dc}/p)_{\min}$ on the value of $(pL)_{\min}$ obtained from our experimental ignition curves measured in a nitrogen discharge by varying the distance L. The solid line $U_{\min} = 407(pL)_{\min}$ and the dashed line $(E_{dc}/p)_{\min} = 407 \pm 5$ V/(cm torr) describe the results obtained fairly well. Hence, for any distance L between the electrodes, the ratio $(E_{dc}/p)_{\min}$ at the minimum of the ignition curve remains constant. This is also true if we change the value of the ion–electron emission coefficient γ of the cathode material (as was noted in [2, 3]).

Note that, by properly choosing the reference axes, we can make all of the obtained ignition curves almost coincide. For example, if we take

$$(pL)^* = pL/(1+(L/R)^2)^a, \tag{1}$$

$$U^*_{dc} = U_{dc}/(1+(L/R)^2)^a, \tag{2}$$

as the abscissa and ordinate, respectively, where $a \approx 0.12$ for nitrogen, then the ignition curves presented in Fig. 1a coincide accurate to ±5 V (Fig. 1b). It is evident that, for $L/R \longrightarrow 0$, we have the usual Paschen curve $U_{dc} = f(pL)$. It follows from Eqs. (1) and (2) that $U^*_{dc}/(pL)^* = U_{dc}/pL = E_{dc}/p$; i.e., the dependences $E_{dc}/p = f((pL)^*)$ for different ignition curves must also coincide (which is seen in Fig. 1b). Here, we also see a reasonable agreement between our results and data from [11, 15–18]. For air, we have $a \approx 0.09$ (Fig. 3) and for hydrogen, we have $a \approx 0.03$ (Fig. 4). Note that, in Figs. 3 and 4, the dependences $E_{dc}/p = f((pL)^*)$ obtained from experimental results [2, 9, 15, 16, 19] agree satisfactorily with our data. From our results, it follows that the scaling law for the gas breakdown can be written in the form $U_{dc} = f(pL, L/R)$ or $U^*_{dc} = f((pL)^*)$.

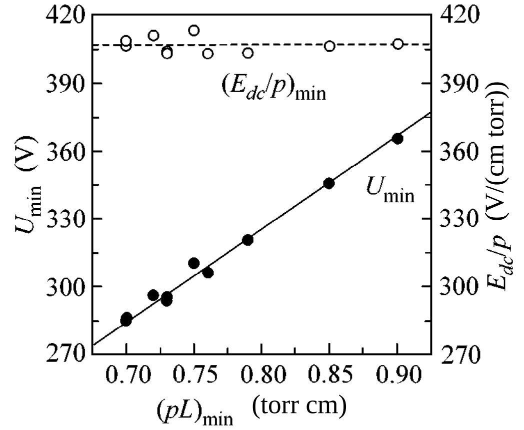

Fig. 2. Dependences of $U_{\min}$ and $(E_{dc}/p)_{\min}$ on $(pL)_{\min}$ for nitrogen. The solid line corresponds to $U_{\min} = 407(pL)_{\min}$, and the dashed line corresponds to $(E_{dc}/p)_{\min}$ = 407 V/(cm torr).

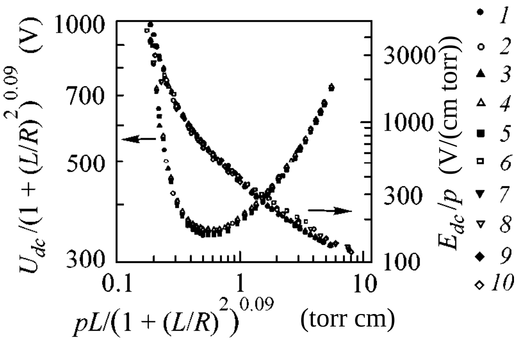

Fig. 3. Dependences of U^*_{dc} and E_{dc}/p on $(pL)^*$ for a glow discharge in air for the interelectrode distances L = (*1*) 0.5, (*2*) 1, (*3*) 2, (*4*) 5, and (*5*) 10 cm and the data from (*6*) [15], (*7*) [19], (*8*) [16], and (*9*) [20], and (*10*) [21].

Based on Eqs. (1) and (2) and the values of breakdown voltage given in the figures, we can calculate, to a high accuracy, the ignition curves for any cylindrical discharge chamber for arbitrary values of the distance

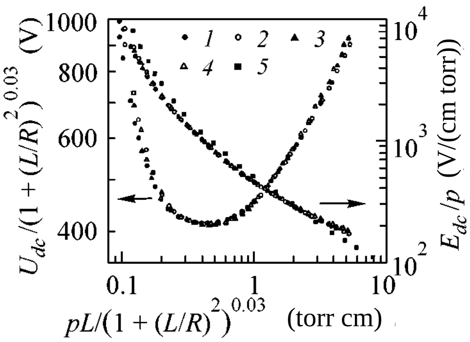

Fig. 4. Dependences of U^*_{dc} and E_{dc}/p on $(pL)^*$ for a glow discharge in oxygen for the interelectrode distances L = (*1*) 0.5, (*2*) 3, (*3*) 6, (*4*) 8 cm and (*5*) the data from [15].

L and radius R. As an example, we find the position of the minimum in the ignition curve for a nitrogen discharge for $R = 3.15$ cm, $L = 5$ cm, and a stainless-steel cathode. From Fig. 1b, it is seen that $U^*_{\min} \approx 280$ V and $(pL)^*_{\min} \approx 0.6$ torr cm. From (1) and (2), we obtain

$$(pL)_{\min} = (pL)^*_{\min}[1 + (L/R)^2]^{0.12}, \quad (3)$$

$$U_{\min} = U^*_{\min}[1 + (L/R)^2]^{0.12}. \quad (4)$$

For $L/R = 1.59$, we have $U_{\min} \approx 323$ V and $(pL)_{\min} \approx$ 0.7 torr cm. From our experimental results, it follows that $U_{\min} \approx 320$ V and $(pL)_{\min} \approx 0.73$ torr cm; i.e., there is a good agreement between the coordinates of the maximum in the ignition curve obtained experimentally and those calculated using Eqs. (1)–(4). To calculate any other point on the ignition curve U^*_1 and $(pL)^*_1$, we use the same procedure. In the general case, to calculate the ignition curve in a cylindrical discharge chamber with arbitrary dimensions, one should have the Paschen curve (i.e., the ignition curve measured in a discharge chamber such that $L/R \longrightarrow 0$ and with the same cathode material). Then, from Eqs. (1) and (2), one can determine the values of U_{dc} and pL. If the initial ignition curve has been measured in a discharge chamber with L_0 and R_0, such that the condition $L_0/R_0 \longrightarrow 0$ does not hold, one should first calculate the dependence $U^*_{dc} = f((pL)^*)$ using Eqs. (1) and (2) and then, from the same relations, calculate the ignition curve for a discharge chamber with the given dimensions L_1 and R_1. This may be written more concisely in the following form:

$$(pL)_1 = pL_0\left[\frac{1 + (L_1/R_1)^2}{1 + (L_0/R_0)^2}\right]^a, \quad (5)$$

$$U_{dc1} = U_{dc0}\left[\frac{1 + (L_1/R_1)^2}{1 + (L_0/R_0)^2}\right]^a, \quad (6)$$

where the index "1" stands for the ignition curve to be sought and the index "0" stands for the initially known ignition curve.

The ignition curve of a glow discharge is usually measured by two methods: (i) the distance L is fixed, and the breakdown voltage U_{dc} is measured at different values of the gas pressure p; and (ii) the value of p is fixed, and the breakdown voltage U_{dc} is measured at different values of the distance L. However, the results obtained in this study show that the latter method of measuring the ignition curve (at a fixed value of p and variable L) is incorrect. The ignition curves obtained in this way are close to the Paschen curve only at small values of L but shift toward higher breakdown voltages with increasing L. Hence, the ignition curve of a glow discharge must be measured by varying the gas pressure p, the distance L between the electrodes being fixed.

In summary, the ignition of a glow discharge in nitrogen, air, and oxygen at a variable interelectrode distance L have been studied experimentally. It is shown that, at any interelectrode distance L, the ratio $(E_{dc}/p)_{\min}$ at the minimum of the ignition curve remains constant. In the range of L/R under study, the scaling law for gas breakdown $U_{dc} = f(pL, L/R)$ holds; i.e., the breakdown voltage U_{dc} is a function not only of the product of the gas pressure p and the gap length L, but also of the ratio L/R. A method for calculating the ignition curve in a cylindrical discharge chamber with arbitrary values of L and R is proposed.

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